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Research article

Control of crop state using remote sensing information

I. M. Mikhailenko

Agrophysical Research Institute RAAS, 14 Grazdansky prospekt, St. Petersburg, 195220 Russia

E-mail: ilya.mihailenko@yandex.ru

Abstract

Theory and methods of use of remote sensing information for making decisions on management of technological operations in precision agriculture are considered. The theory can be applied for optical remote sensing systems based on space satellites, aviation vehicles, and agricultural machines.

Keywords: remote sensing, measurement, evaluation, crop and soil state, mathematical models, off-line and on-line control

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Introduction

A rapid development of remote sensing (RS) tools based on space satellites, aviation vehicles, and agricultural machines has required an accelerated development of theory of operative management of crop and soils state using the RS information. The theory must effectively use possibilities of the RS information and ensure the highest level of automation of management of the crop and soil state. Main stages of the management of the crop and soil state and its spatial heterogeneity were considered in papers (Mikhailenko, 2005; Rachkulik, Sitnikov, 1981; Krishchenko, 1997; Pyt'ev, P. 2004). These stages include: (a) identification of mathematical models of the crop and soil state; (b) identification of models of tools for measuring the crop and soil state using the RS information; (c) selection of control's goal and optimality criterion; (d) development of algorithms of off-line and on-line control; (e) synthesis of executive on board regulator.

A key problem in the fulfillment of these stages is an adaptation of the mathematical models to actual changes in the crop and soils states. The adaptation of mathematical models includes a periodic assessment of model parameters using actual errors of simulation. Therefore there is a need to collect the plant samples twice a week uniformly on entire agricultural plots. This work is, firstly, very time and labor expensive on large-scale plots and, secondly, significantly reduces the degree of automation of management process. It is proposed to use small reference plots which (1) are located near main agricultural plots and (2) have similar management practices with several levels of technological impacts. This approach allows to distinguish the significant changes in the crop and soil states and to increase the accuracy of identification of mathematical models of the RS measurements. Such mathematical models are universal because they are based on the basic physical laws of reflection of non-continuous media. Therefore these models can be used for a rapid evaluation of the crop and soil state on main agricultural plots.

Materials and Methods

Formulation of optimization problem

A starting point in any system of optimal management is the selection of its goal and adequate criterion of optimality. The goal of optimal management is being formulated according to: (1) the requirements to final results and (2) the presence of the resource and environmental constraints. In agriculture, the goal of optimal management is to obtain a given profit per unit area under the pre-determined environmental constraints. In order to justify: (1) the form of criterion of optimality, (2) the type and the shape of mathematical models, the following crop and soil parameters are being selected in:

– model compartment of the state of perennial grasses (Mikhailenko, Kurashvili, 2008; Mikhailenko, Timoshin, Danilova, 2009): x_{1m} – total mass per unit area, kg m^{-2}; x_{2m} – dry mass per unit area, kg m^{-2}; x_{3m} – nitrogen content in plants, mg kg^{-1}. All these parameters are combined into a dimensional vector [3×1] – X_m;

– model compartment of the soil state: x_{1s} – soil moisture content in root zone, kg kg^{-1}; x_{2s} – content of available nitrogen, g kg^{-1}; x_{3s} – content of available potassium, g kg^{-1}; x_{4s} – content of available phosphorus, g kg^{-1}. All the parameters are combined into a vector of dimension [4×1] – Xs;

– variables of management: u_1 – output of irrigation water, kg m^{-2}; u_2 – rate of nitrogen fertilizers, g m^{-2}; u_3 – rate of potassium fertilizers, g m^{-2}; u_4 – rate of phosphate fertilizers, g m^{-2}. All the parameters are combined into a total vector of management [4×1] – U;

– climate variables: f_1 – daily air temperature, °C; f_2 – daily rainfall, mm; f_3 – daily radiation, W m^{-2}. All the variables are combined into a vector of external disturbances [3×1] – F.

– variables of cost: c – cost of unit harvest; C – vector of costs for resource management.

Models of optical measurements

If there is a need to effectively use the spectral optical channels in algorithms of assessing the state of the soil – plant – atmosphere system, informative indicators for each channel are being used. These indicators are ranked in size, and measuring system consists of channels which have a maximum information content. As a result of such a selection, two combined sub-vectors, namely, Y_X – sub-vector of optical parameters of the crop state and Y_G – sub-vector of optical parameters of the soil state are being created from the above-mentioned Y vector. These sub-vectors can have the same components as many reflectance optical parameters are simultaneously related to both crop and soil state.

Based on the basic physical laws of light reflection from an inhomogeneous surface, including plants, the model of measurements can be represented in the following vector-matrix symbolic form (Neustadt at all, 2006; Rachkulik, Sitnikov, 1981):

$$Y_X = \mathbf{p}e^{-P^T X_\mathbf{m}} + E_\mathbf{m}, \tag{1}$$

where: p – a vector of scale parameters; P – matrix of parameters between the state of crops and the most informative optical parameters; X_m – the dimensional vector; E_m – a vector of random errors of simulation; T – index vector and matrix transpose.

If all the parameters of the model (1) are known, current estimates of the crop state are being determined by the measured vector of optical parameters which are quantified by solving the following optimization task:

$$\widehat{X} = \arg\min(Y_X - \mathbf{p}e^{-P^T\widehat{X}})^2 . \tag{2}$$

The following linear model can be applied for the soil state:

$$Y_G = HX_S + E_S, \tag{3}$$

where Y_G – sub-vector of optical parameters of the soil state at different optical ranges; G – vector of the soil state's parameters; H – matrix of parameters quantified on the basis of experimental results; E – vector of random errors of measurements of optical parameters at zero mathematical expectation and covariance matrix R. In the model (3) the current estimates of the measured soil state are being determined by (Pyt'ev, 2004):

$$\widehat{X}_\mathbf{s} = \left[R^{-1/2}H^T\right]^{-1} R^{-1/2}Y_G . \tag{4}$$

Operative estimates (4 and 6) are used to carry out the adaptation of the dynamic models of crops and soil state.

Models of crop and soil state

All the above-mentioned techniques can be used for average crop and soil states. However, the RS tools enable to use the whole picture of spatial distribution of the measured parameters and the related crop and states as variables. The presence of adaptation procedure enables to apply the linear models in the following form (Mikhailenko, 2005):

$$\begin{aligned}&\dot{X}_m(z,h) = A_m X_m(z,h) + C_m F(t,z,h) + B_m X_s(t,z,h) + \xi(t,z,h),\\ &t \in (t_0,T),\ X_m(t_0,z,h) = X_{m,0}(z,h);\end{aligned} , \qquad (5)$$

where: $X_m^T = \begin{bmatrix} x_{1m} & x_{2m} & x_{3m} \end{bmatrix}$ - the state vector model; ξ - [3x1] vector of random modeling errors, taking into account all sources of uncertainty in the model;

$A_m = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_m$ - dynamic matrix, $C_m = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}_m$ - matrix perturbation,

$B_m = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ 0 & 0 & b_{23} & b_{24} \\ b_{31} & b_{31} & 0 & 0 \end{bmatrix}_m$ - control matrix; z, h - the spatial variables.

Model of the state of the soil environment has a similar structure

$$\begin{aligned}&\dot{X}_s(z,h) = AX_s(z,h) + C_s F(t,z,h) + MX_m(t,z,h) + B_s U(t,z,h),) + \zeta(t,z,h),\\ &t \in (t_0,T),\ X_s(t_0,z,h) = X_{s0}(z,h),\end{aligned} , \qquad (6)$$

where: $X_s^T = \begin{bmatrix} x_{1s} & x_{2s} & x_{3s} & x_{4s} \end{bmatrix}$ - the state vector model; ζ - [4x1] vector of random noise in the model;

$\mathbf{A}_s = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_s$ - the dynamic matrix, $C_s = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{bmatrix}_s$ - matrix perturbation,

$B_s = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}_s$ - matrix control, $M_s = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{43} \end{bmatrix}_s$ - the coupling matrix model of the structure of biomass, the model state of the soil environment.

Identification of the model of measurements

The task of operative adaptation of model of an optical measurement (1) is to estimate the vector **p** and the matrix **P** using plant biomass and soil samples collected from the reference plots during the growing season. As a result of sampling and simultaneous measurements of the optical parameters, we obtain an increasing massive amount of data (average for plot) as {Y_X [k], X_M [k]}, where – k - number of estimation step. Based on the massive amount of data it is necessary to quantify the parameters of model of measurement (1). This task is realized through the following optimization procedure:

$$\hat{\mathbf{p}}, \hat{P}[k] = \underset{\hat{\mathbf{p}}, \hat{P}[k]}{\arg\min} \left(Y_X[k] - \mathbf{p}[k] e^{-\hat{P}^T[k] X_m[k]} \right)^T \left(Y_X[k] - \mathbf{p}[k] e^{-\hat{P}^T[k] X_m[k]} \right). \qquad (7)$$

Similar massive amount of data is generated for the model of measurement of the soil state (6) – {Y_G [*k*], X_s [*k*]}. An identification algorithm of this model provides a minimum mean square error:

$$\mathrm{H} = \arg\min_{\mathrm{H}} \left[\mathrm{Y_G} - \mathrm{HX}_S\right]^{\mathrm{T}}\left[\mathrm{Y_G} - \mathrm{HX}_S\right]. \tag{8}$$

Identification of model of crop and soil state

An operative adaptation of the crop and soil state's model to the reference plots allows to create the estimates of crop $\widehat{\mathrm{X}}_{\mathrm{M}}(t)$ and soil $\widehat{\mathrm{X}}_s(t)$ state using the current optical measurements $\mathrm{Y_X}$, $\mathrm{Y_G}$ and algorithms (2, 4). Such a procedure is possible because the models of optical measurements (1) and (3) are universal for the reference and main agricultural plots. Nevertheless, the dynamic models of crop and soil state of crops are not universal for the reference and main agricultural plots. Therefore the dynamic models need to be adapted to current management and environmental conditions. To fulfill the adaptation of the dynamic models, a set of average data ({ $\widehat{\mathrm{X}}_{\mathrm{M}}(t)$, $\widehat{\mathrm{X}}_{\mathrm{s}}(t)$, F(*t*), U(*t*)}) is being formed:

$$\mathrm{A_m,A_s,B_m,B_s,C_m,C_s,M} = \arg\min_{\mathrm{A_m,A_s,B_m,B_s,C_m,C_s,M}} \int_{t_0}^{t} \{g_1[((m_{\mathrm{X_m}}(t) - \widehat{\mathrm{X}}_{\mathrm{m}}(t))^{\mathrm{T}}$$
$$((m_{\mathrm{X_m}}(t) - \widehat{\mathrm{X}}_{\mathrm{m}}(t))] + + g_2[((m_{\mathrm{X_s}}(t) - \widehat{\mathrm{X}}_{\mathrm{s}}(t))^{\mathrm{T}}((m_{\mathrm{X_s}}(t) - \widehat{\mathrm{X}}_{\mathrm{s}}(t))]\}dt, \tag{9}$$

where – g_1, g_2 – weight multipliers of adaptation criterion (7), which enable to determine the ratio of errors in the crop state and soil state adaptation; $m_{\mathrm{X_m}}$, $m_{\mathrm{X_s}}$ – modeled vectors of the crop and soil state.

Programmed component of off-line control

The profit per unit area is determined as follows:

$$\Pi(T) = c\mathrm{H^T}\mathrm{X}_{\mathbf{m}}(T), \tag{10}$$

where: $\mathrm{H^T}$ = [0 1 0] – output matrix which is used quantify the net output of crop yields.

Using the value of a given profit Π*, a criterion management optimality, i.e. an achievement of required profit per unit arable area, is being determined as:

$$I = [\Pi^* - c\mathrm{H^T}\mathrm{X}_{\mathbf{m}}(T)]^2 + \mathrm{C^T}\int_{t_0}^{T}\mathrm{U}(t)dt \xrightarrow[\mathrm{U}(t)\in\Omega_{\mathrm{U}}]{} 0, \tag{11}$$

where Ω_{U} – a range of threshold management measures from an environmental point of view.

A large size of management plot and a limitation in resources resulted in solving the control component by two steps (Mikhailenko, 2005). Firstly, an optimal program of changes in the vector (as a factor of control) of crop and soil state is created, whereas this program is considered to ensure an achievement of potential crop yield. Secondly, types of technological operations are being identified to relate stronger the soil state to the optimal program. In this case, initial criterion of optimality is divided into two sub-criterions:

$$I_1 = [\Pi^* - c\mathrm{H^T}\mathrm{X}_{\mathbf{m}}(T)]^2 \xrightarrow[\mathrm{X_s}(t)]{} 0, \tag{12}$$

$$I_2 = \int_{t_0}^{T}\{[(\mathrm{X_S}(t) - \mathrm{X_S^*})^{\mathrm{T}}(\mathrm{X_S}(t) - \mathrm{X_S^*}(t))] + \mathrm{C^T}U(t)\}dt \xrightarrow[\mathrm{U}(t)\in\Omega_{\mathrm{U}}]{} 0. \tag{13}$$

As a result of the minimization procedure (12), the optimal program of changes in the vector of soil state $\mathrm{X_S^*}(t)$ is being formed. The optimal program ensures the achievement of the potential crop yields without considering the resource limitations. The program is also used for solving the parameter optimization of technological operations. Both degree technological impacts and time of their realization are considered as parameters of optimization of technological operations. In the agricultural technologies, the management impacts are usually linked to phenological phases of plants. In this case, a whole growing season can be divided into inter-phase periods. In the case of perennial grasses, the whole growing season is being divided into the periods between cuttings, while the technological operations are both irrigation and nitrogen fertilization events at the beginning of each of the periods between cuttings. The optimal rates of irrigation water and nitrogen fertilizers are to be a result of minimization of criterion (13) regarding the optimum program of crop development $\mathrm{X}_m^*(t)$.

Component of on-line control

A key aim of real-time control of technologies is to: (i) apply the off-line optimal program for entire agricultural plots and (ii) carry out its correction according to a spatial heterogeneity in the crop and soil states. In the precision agriculture, the on-line and off-line components are strongly linked, in contrast to classical knowledge on control, in order to support a principle of total optimization of technologies. This optimal way of real-time control is fulfilled by a regulator of technological machines according to the following law of control (Mikhailenko, 2005):

$$\mathrm{V}(z,h|t_i) = \mathrm{v}^*(t_i) + \mathrm{K}_1^{\mathrm{T}}(\widehat{\mathrm{X}}_m(z,h|t_i - \mathrm{X}_m^*(t_i)), \qquad (14)$$

where $\mathrm{v}^*(t_i)$ – the programmed component of off-line control for the entire area of agricultural plots; $\mathrm{K}(\widehat{\mathrm{X}}_m(z,h|t_i) - \mathrm{X}_m^*(t_i))$ – a component for a compensation of spatial heterogeneity of the crop states $(\breve{\mathrm{X}}_m(z,h|t_i)$, using the RS estimates of the crop states and the program of optimal crop development $\mathrm{X}_m^*(t_i)$; K – matrix controller converting signal deviations into corrections in control; *i*- index of elementary plots control.

Results

The results of use of programmed off-line component of control for perennial grasses are presented in Fig. 1. Solid lines and shaped points show the potential and programmed crop yields, respectively.

Fig. 2 shows the changes in content of soil available nitrogen and moisture for optimal technological operations. The shaped straight lines reflect the levels of soil parameters at the potential yield of perennial grasses, whereas the solid lines show a dynamics in soil parameters during the growing season of perennial grasses.

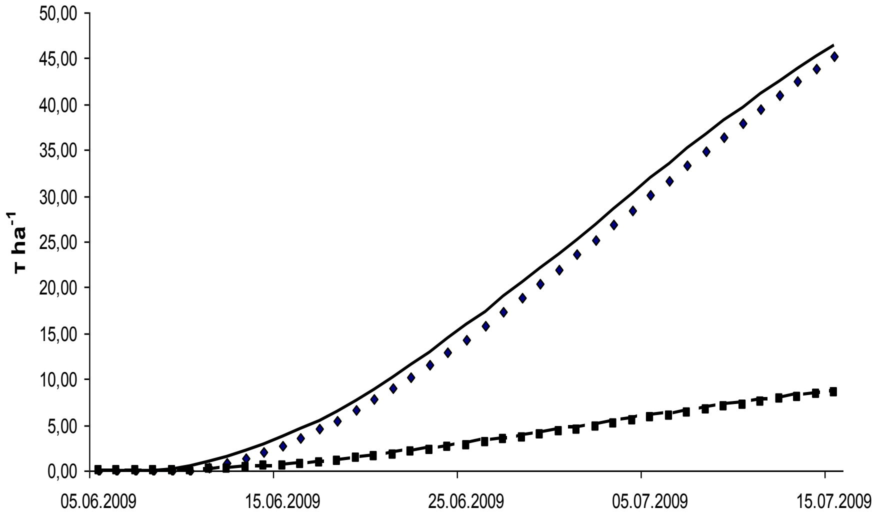


Figure 1: The dynamics of accumulation of potential and programmed crop yield at a given perennial grass yield of 40 t ha^{-1}

◆ – biomass potential; ■ – dry matter potential;

— – biomass programmed**;** - - - –dry matter programmed

Fig. 2 shows the changes in content of soil available nitrogen and moisture for optimal technological operations. The shaped straight lines reflect the levels of soil parameters at the potential yield of perennial grasses, whereas the solid lines show a dynamics in soil parameters during the growing season of perennial grasses.

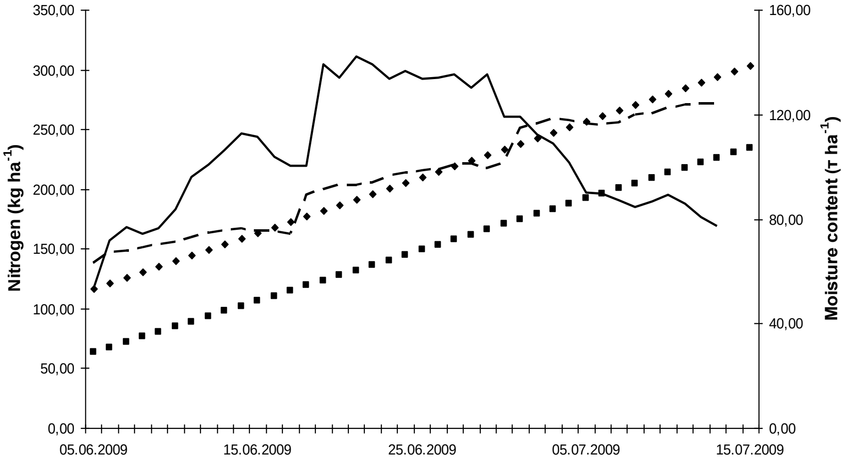


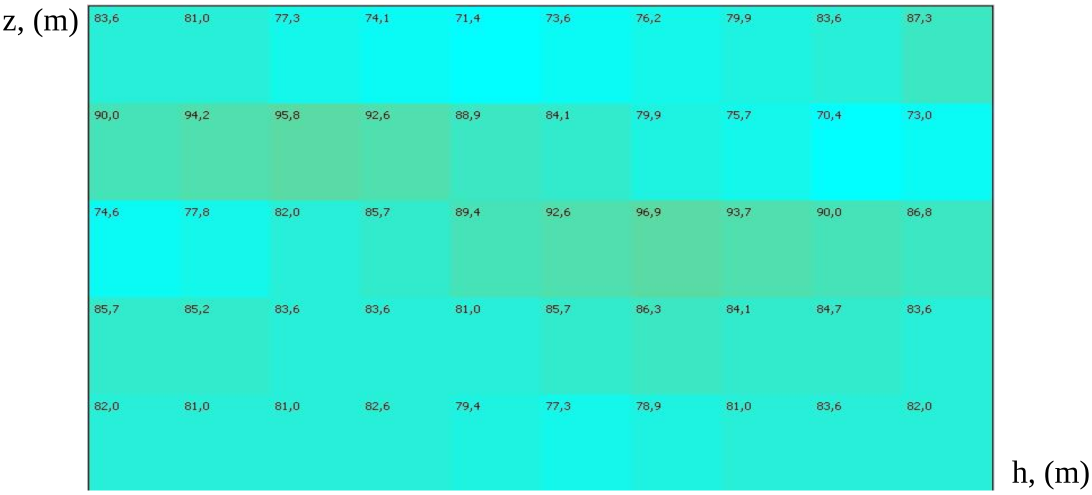
Figure 2: The dynamics of soil available nitrogen and moisture content
■ – nitrogen potential;◆ – moisture content potential;
— – nitrogen programmed - - – moisture content programmed

Table 1 shows rates of nitrogen fertilizers and irrigation water applied in the framework of optimal technological operations.

Table 1: Parameters of optimal technological operations

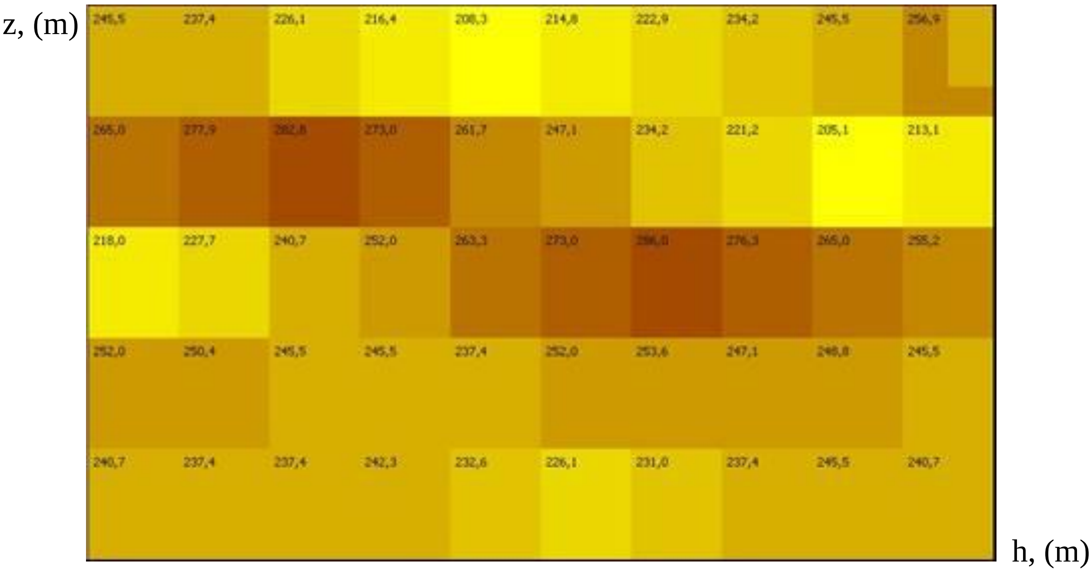
	Parameters	
Date	**Rate of nitrogen fertilizers, kg ha^{-1}**	**Rate of irrigation water, t ha^{-1}**
5 June 2011	0	31.5
18 June 2011	65.0	250.0
01 July 2011	0	350.2

Fig. 3 and 4 show a picture of spatial distribution of optimal rates of nitrogen fertilizers and irrigation water on the plot with perennial grasses. An area of each management plot is 1.8–2.0 m^{-2}. A realization of such a site-specific management was carried out by agricultural machinery developed at the Agrophysical Research Institute.



83,6	81,0	77,3	74,1	71,4	73,6	76,2	79,9	83,6	87,3
90,0	94,2	95,8	92,6	88,9	84,1	79,9	75,7	70,4	73,0
74,6	77,8	82,0	85,7	89,4	92,6	96,9	93,7	90,0	86,8
85,7	85,2	83,6	83,6	81,0	85,7	86,3	84,1	84,7	83,6
82,0	81,0	81,0	82,6	79,4	77,3	78,9	81,0	83,6	82,0

Figure 3: The spatial distribution of optimal rates of nitrogen fertilizers
(Fragment of the field)



245,5	237,4	226,1	216,4	208,3	214,8	222,9	234,2	245,5	256,9
265,0	277,9	282,8	273,0	261,7	247,1	234,2	221,2	205,1	213,1
218,0	227,7	240,7	252,0	263,3	273,0	286,0	276,3	265,0	255,2
252,0	250,4	245,5	245,5	237,4	252,0	253,6	247,1	248,8	245,5
240,7	237,4	237,4	242,3	232,6	226,1	231,0	237,4	245,5	240,7

Figure 4: The spatial distribution of optimal rates of irrigation water
(Fragment of the field)

Approbation

The mathematical models of optical measurements were developed on the basis of results of identification experiments at the Menkovo experimental station of the Agrophysical Research Institute in 2009–2011. The mathematical models of the state of perennial grasses were developed on the basis of experimental data obtained on agricultural plots of two farms in St. Petersburg region in 2004–2011. These agricultural plots were also used to develop the algorithms for optimal control of state of perennial grasses after their three cuttings in 2009–2010.

Conclusion

The proposed theory of control of crop and soil state is developed on the basis of modern basic knowledge, which reflects such important features of control object as its size, nonstationarity, hyper inertia, and spatial distribution. The proposed new mathematical models and algorithms of assessment and control of the soil and crop state are based on detailed information obtained by remote sensing tools and software-equipped technological machinery which is able to fulfill the technological operations on 1.5–2.0 m^2 plots.

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